Fermi National Accelerator Laboratory

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A Prototype Lattice Design for a 2 TeV μ^+—μ^- Collider

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A Prototype Lattice Design for a 2 TeV $\mu^+-\mu^-$ Collider

This is a description of a prototype lattice for a $\mu-\mu$ collider[1]. It is not intended to be a final design, that is the design for a collider lattice which will be built. This prototype design will need to be optimized as the constraints and the requirements for the collider are better understood. As this is done the present design will be replaced by a superior design. Nonetheless this prototype can be used as a benchmark and a means of identifying problems.

This is a geometric design, that is the beam energy does not enter into the properties of the elements of the design. Since the proposed energy for the $\mu-\mu$ collider is 2TeV x 2TeV this energy has been used to compute the gradients needed in the quadrupoles. At an energy of 2TeV, and with superconducting dipoles of ~8T the circumference of the ring is ~6km. Since the luminosity depends exponentially on the length of the lattice an important design criterion will be to minimize the length of the ring. This will be done with careful design of the interaction regions (IR), the utility straight sections, and even the standard cells in the arcs.

This model design has been constrained as follows:

- It uses magnets which are reasonable (factors of 2 in the gradients) extrapolation from existing superconducting magnets;
- The value of the lattice function at the interaction point, β^*, be 0.003m (3mm) in both planes;
- The dispersion, η, at the interaction point be zero;
- The lattice has two low-β IRs and two long high-β utility straight sections;
- The lattice functions at the end of the insertions, low-β and high-β, be matched with the lattice functions in the arcs.

The arcs will be built of FODO cells, the "standard cell".

The most direct way to have zero dispersion at the IP is to have zero dispersion throughout the IR. This is most simply done with dispersion killers before the insertions. The design of a dispersion killer is most easily done if the phase advance of the standard cell is either 60° or 90°. A phase advance of 60° has been chosen for the standard cell.

The other parameter used to define the standard cell is β_{max}. In principle this is arbitrary but in order to match the low-β insertion into the arcs I found it better to use a relatively low β_{max} of 75m. A more elaborate IR would make it be possible to use a larger value. The advantage of a larger value for β_{max} might be to increase the packing fraction, the percentage of the ring filled with dipoles, and hence reduce the cost. With a 75m β_{max} the packing fraction in the standard cell is of the order of 80-90% depending on the space needed for correction elements.

The design of a utility straight section is easily done using the idea of matching quads due to Tom Collins. The value of the lattice function β at the middle of the straight has been set at 200m but the value can be changed within reasonable limits. Within the utility straight the maximum β is ~300m. The dispersion across the utility straight is matched by having dispersion killers at each end of the straight. This, of course, lengthens the ring, and if a zero dispersion straight is not needed then perhaps a different design could be developed. This alternate design, if it had any value, would need to be shorter and would also need to match the dispersion across the straight without making it zero.

The design of the low-β IR, matching it to the arcs, is the most critical part of the design. A solution has been found. It is *not* optimal. It is a proof of principle, not a final design. The parameters of the quadrupoles in the IR, in the utility straight sections, and in the arcs are shown in Table I. The values for the β function for the different parts of this lattice are plotted in figures 1, 2 and 3.

The matching of the IR values of the lattice function to the arcs is very sensitive to the precise values of the gradients in the low-β quadrupoles. Figures 4, 5 and 6 show the values of the β functions for the same parts of the lattice as in figures 1, 2, and 3 except that now the gradient of the middle member of the final triplet upstream of the IR has been increased by 2×10^{-5} from the design value. A large β wave is seen in the arcs. The values of the lattice functions at the IP have also changed. The calculated values are found in Table II. With an error of 2×10^{-6} the β wave in the arc is negligible.

The tunes for this lattice depend on the number of FODO cells in the arcs and can be adjusted with correction quadrupoles. The natural chromaticity, that is the chromaticity with no sextupole correctors, is very large with $\zeta_x \sim \zeta_y \sim -3500$. The large value for the chromaticity comes from the low-β insertions. We can correct this chromaticity with correctors in the arcs in which case we need very strong sextupoles $B''/[B\rho] \sim 4$ to be compared with the values, $B''/[B\rho] \sim 4 \times 10^{-2}$, typically used in the Tevatron to correct and control the chromaticity. Sextupoles in the insertions cannot be used to correct the chromaticity because of the zero dispersion in the insertions. High dispersion regions could be constructed if that proves to be necessary.

The values of the lattice functions have also been computed for a momentum offset, $\delta p/p$, of 1×10^{-5}. The results are plotted in figures 7, 8, and 9. Again we see a large β wave in the arcs. Since we will require a large $\delta p/p$ it will be necessary to design the IR so that we do not have these large β waves in the arcs for off momentum particles.

The prototype lattice discussed here, without chromaticity correction, has no non-linear elements. The stable region will have a non-zero value for the non-integer part of the tune. With a chromaticity with absolute value of 3500 this yields a full width for the momentum aperture of ~1/3500 or $\sim 2.8 \times 10^{-4}$. This has been confirmed with a tracking code.

With the chromaticity correcting sextupoles third integer values of the tune become unstable as well as integer values. In addition the chromaticity is now non- linear. Tracking calculations indicate that the momentum aperture is reduced by perhaps a factor of 2 with the addition of the strong sextupoles due to the existence of the 1/3 resonance and the non-linear chromaticity remaining even when the linear chromaticity is corrected with the sextupoles.

Dynamic Aperture.

For the linear lattice the dynamic aperture is given by the maximum value of β and the size of the physical aperture. At the maximum value of β in the lattice, which occurs in the inner triplet, of $\sim 2 \times 10^5$m (200km) and a physical aperture of ±4cm, the normalized *admittance* is ~150πmmmr. To have this admittance outside of the low-β insertion would require an aperture with only a ~.7mm diameter. This is clearly smaller than any practical aperture in the lattice. The dynamic aperture, in the case of this linear lattice, should not be important in determining the aperture in the arc dipoles and quadrupoles. The size of the dynamic aperture has been confirmed by tracking calculations.

The dynamic aperture is not restricted by the values of the chromaticity sextupoles.

The lattice described here is an ideal lattice, a lattice whose components have precisely the strengths given by the lattice design code.This, to say the least, is unrealistic. I have not attempted any calculations based on a model with magnet errors. This can, of course, be done but at this time seems to me to be premature. One point is interesting however. The μ–μ collider is the only circular collider, that I know of, where one can track for the full life time of the beam (~1000 turns) using a reasonable amount of computer time.

Conclusion

A prototype lattice for a μ–μ collider has been constructed. The dynamic aperture is determined, for reasonable values of the bore of the dipoles and the quadrupoles in the arcs, by the large value of the maximum β in the low-β insertion. The momentum aperture is determined first by the chromaticity of the lattice, and if the chromaticity is corrected by sextupoles, by the third order resonance. Chromaticity correction will require very strong sextupoles unless regions of high dispersion are created.

References:

1. A High -Energy High-Luminosity μ^+-μ^- Collider **David V. Neuffer(CEBAF) Robert Palmer** (BNL) *BNL-61267,CAP109-MUON-94C*

```
! Program run at  9:28:49   on 13Mar-1995
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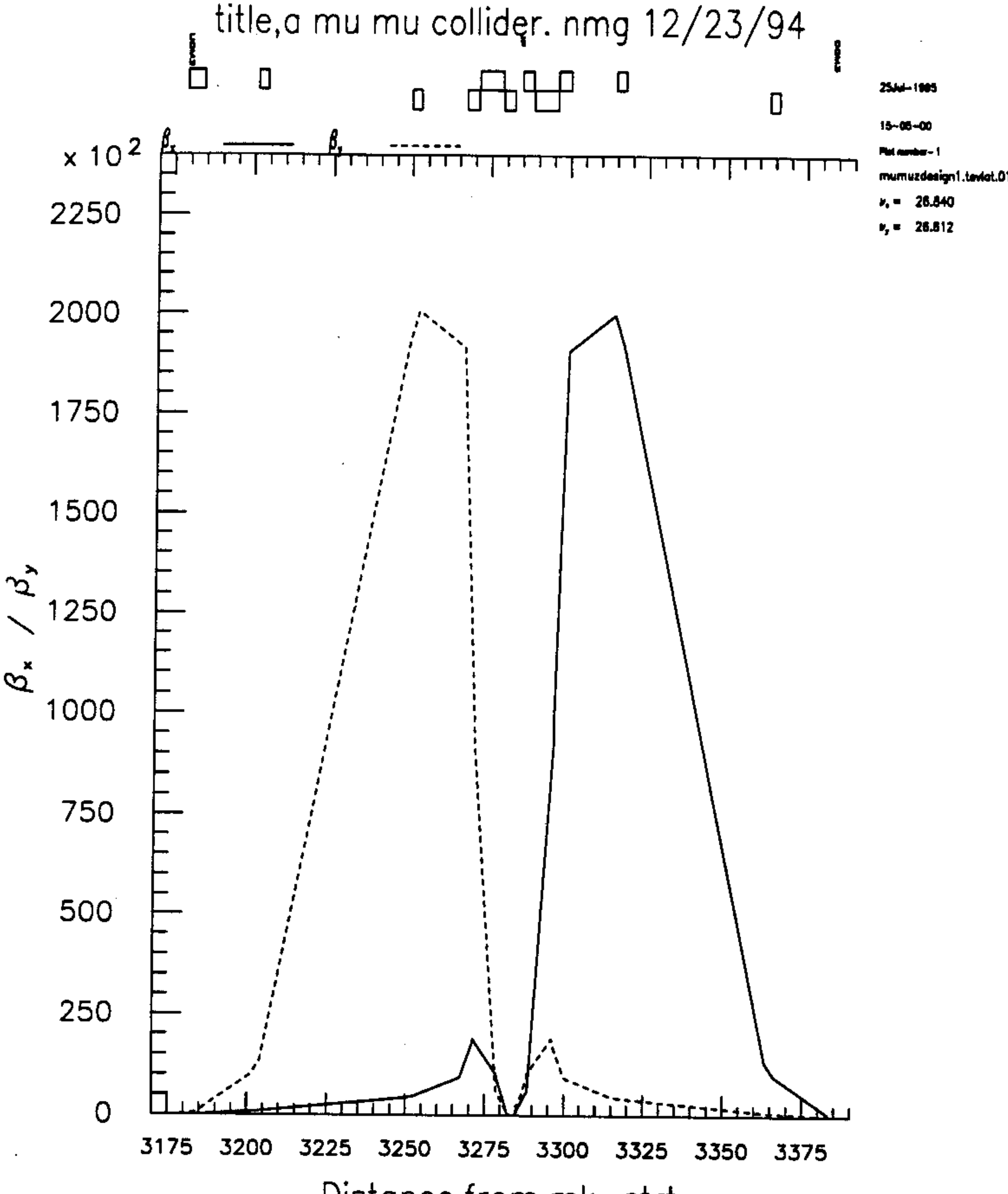
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2000
1750
1500
1250
1000
750
500
250
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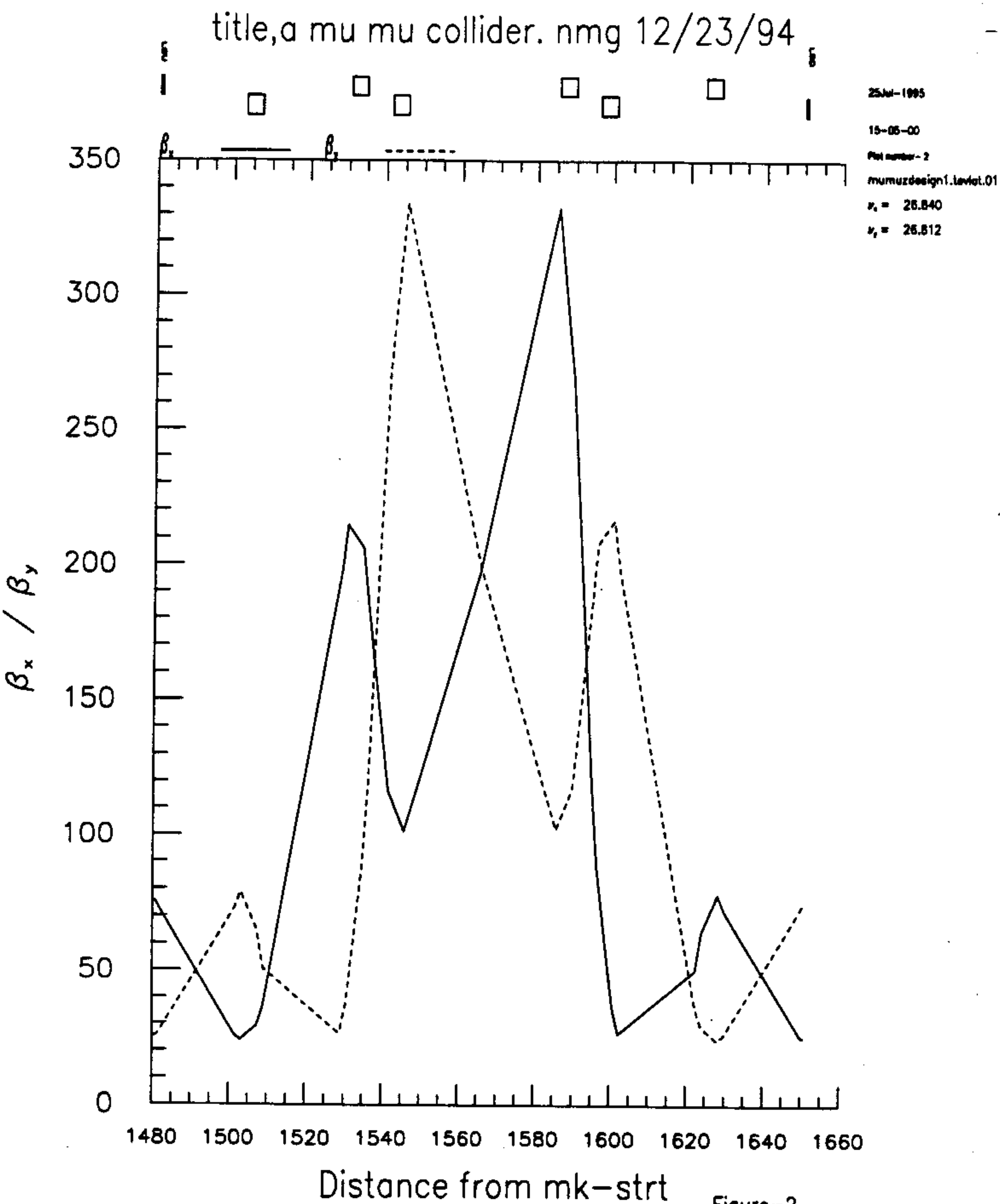
3200
3225
3250
3275
3300
3325
3350
3375
Distance from mk-strt

Figure-1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ν_y = 26.812
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300
250
200
150
100
50
0
β_x / β_y
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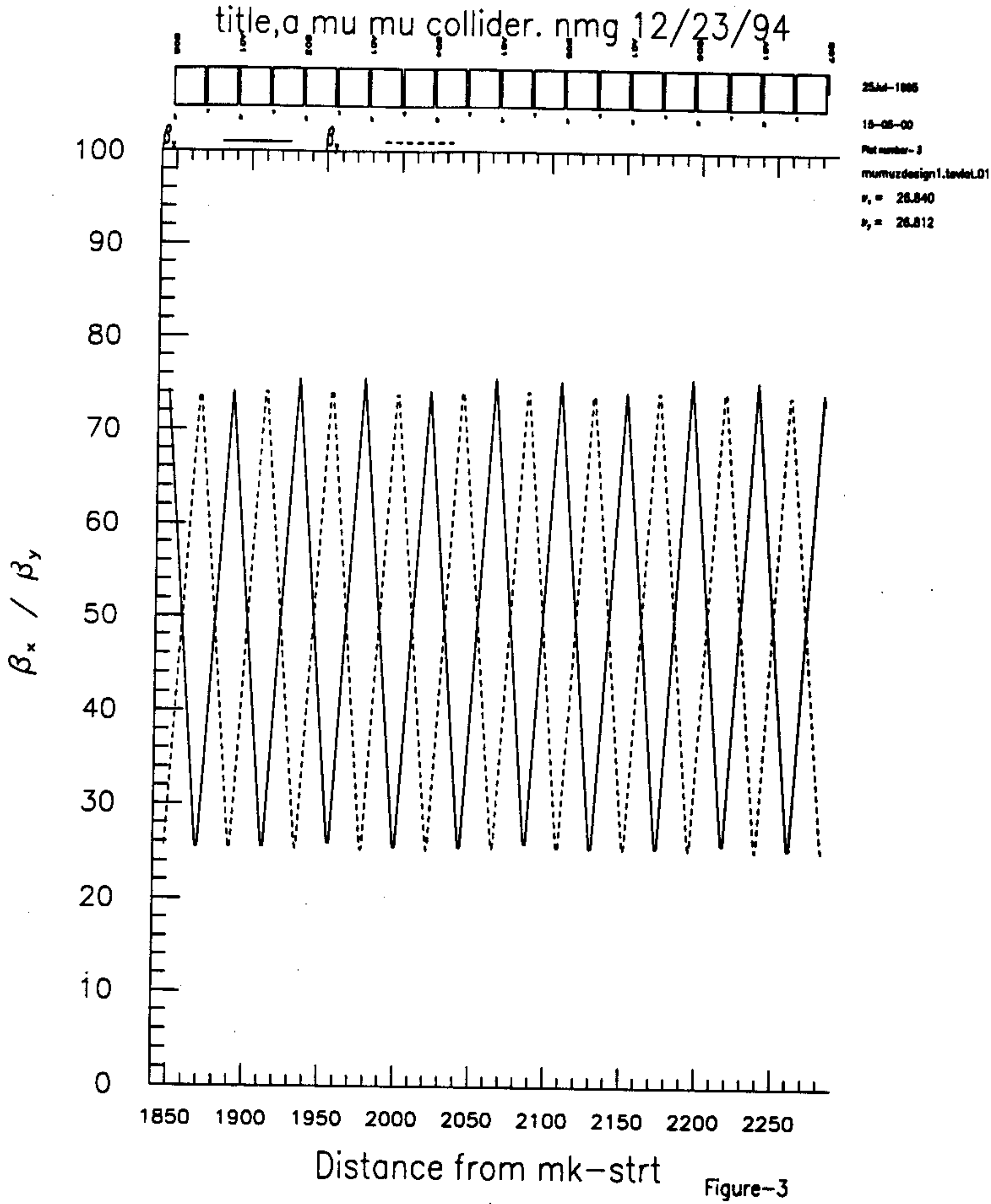
1500
1520
1540
1560
1580
1600
1620
1640
1660
Distance from mk−strt

Figure−2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β_y
β_x / β_y
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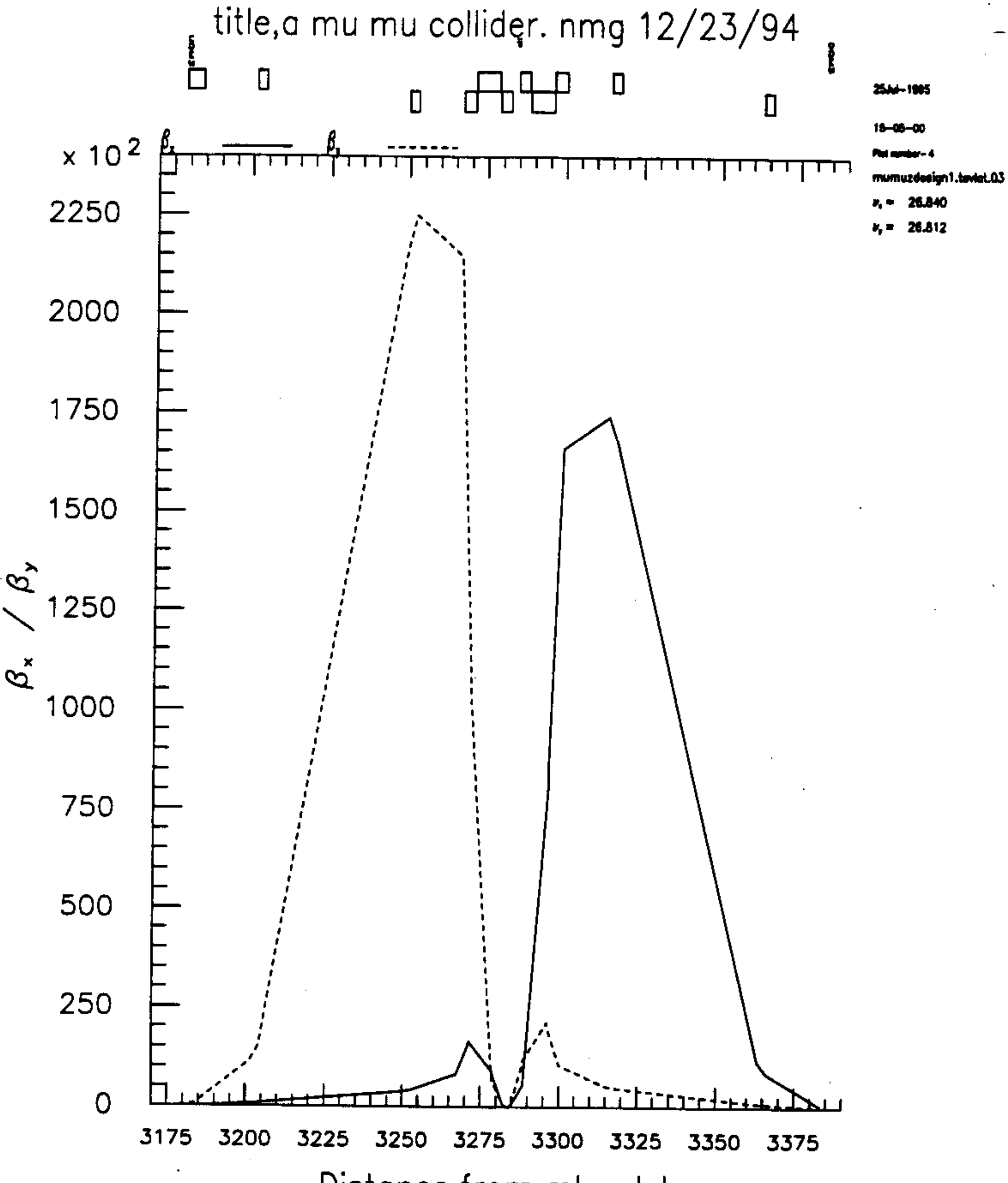
20
30
40
50
60
70
80
90
100
1850
1900
1950
2000
2050
2100
2150
2200
2250
Distance from mk-strt

Figure-3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2000
1750
1500
1250
1000
750
500
250
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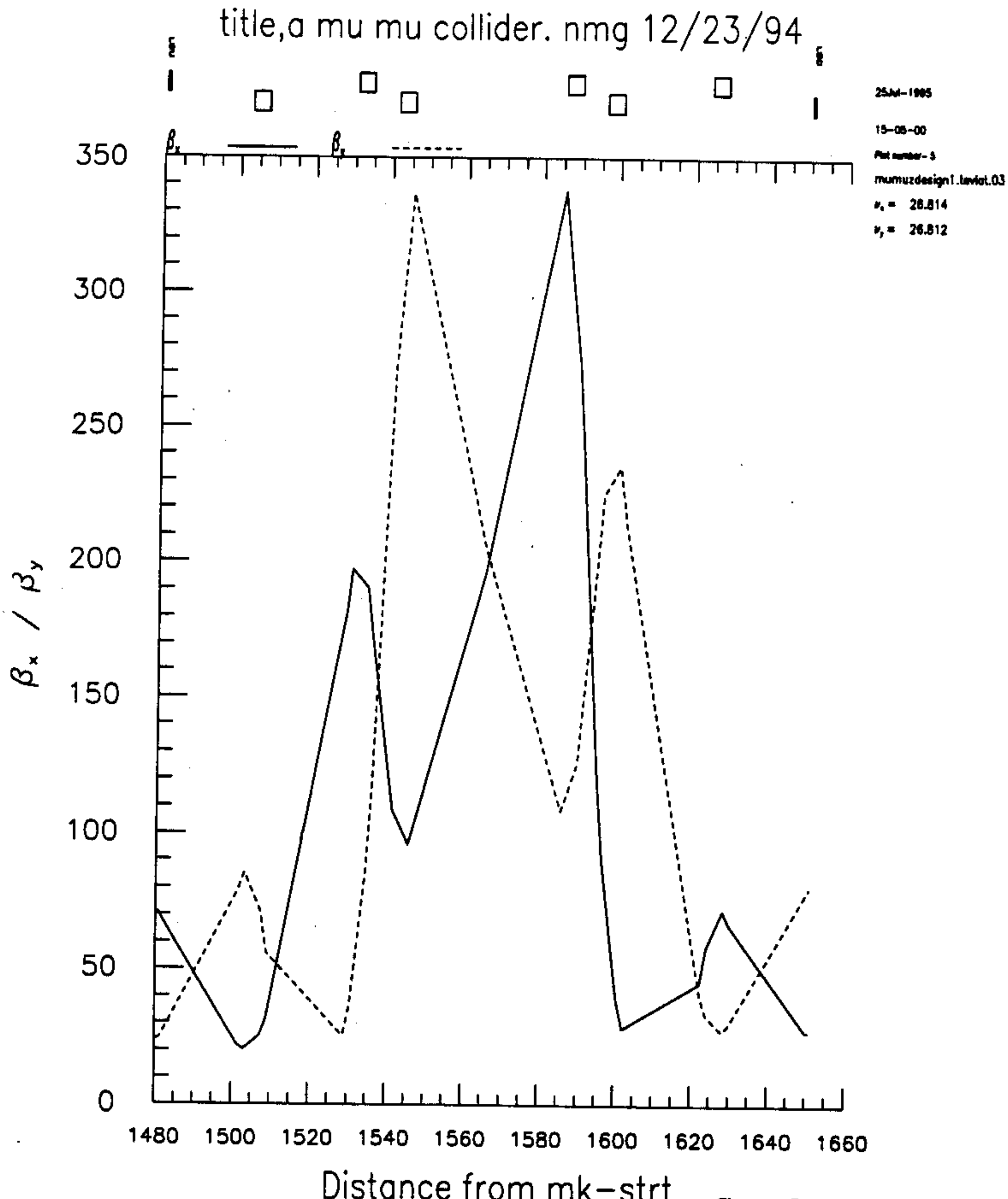
βx / βy
3175
3200
3225
3250
3275
3300
3325
3350
3375
Distance from mk-strt
26.840
26.812

Figure-4



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β_y
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300
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200
150
100
50
0
β_x / β_y
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1500
1520
1540
1560
1580
1600
1620
1640
1660
Distance from mk-strt
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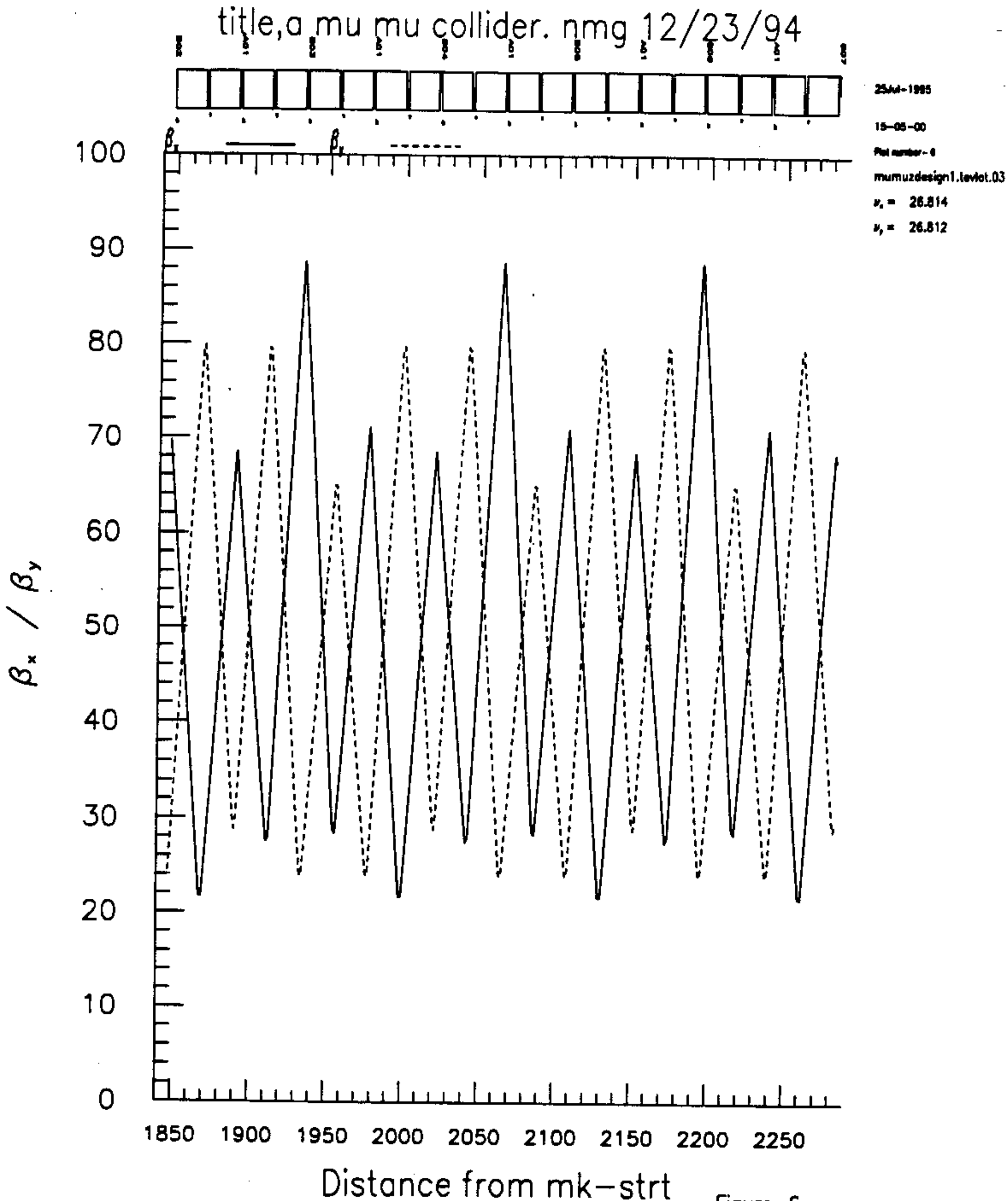
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β_y
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90
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20
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β_x / β_y
1850
1900
1950
2000
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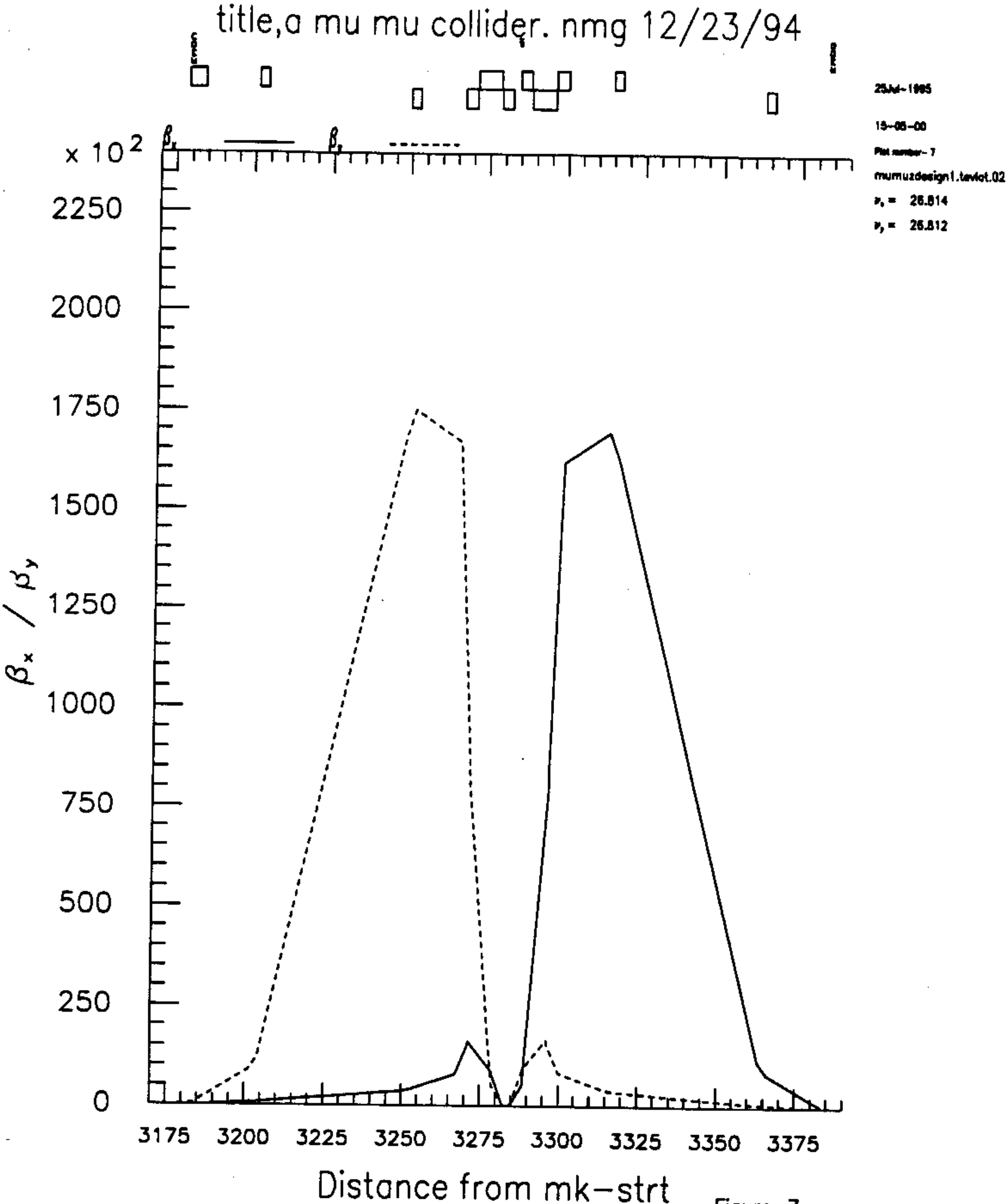
2150
2200
2250
Distance from mk-strt

Figure-6



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2000
1750
1500
1250
1000
750
500
250
0
β_x / β_y
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3200
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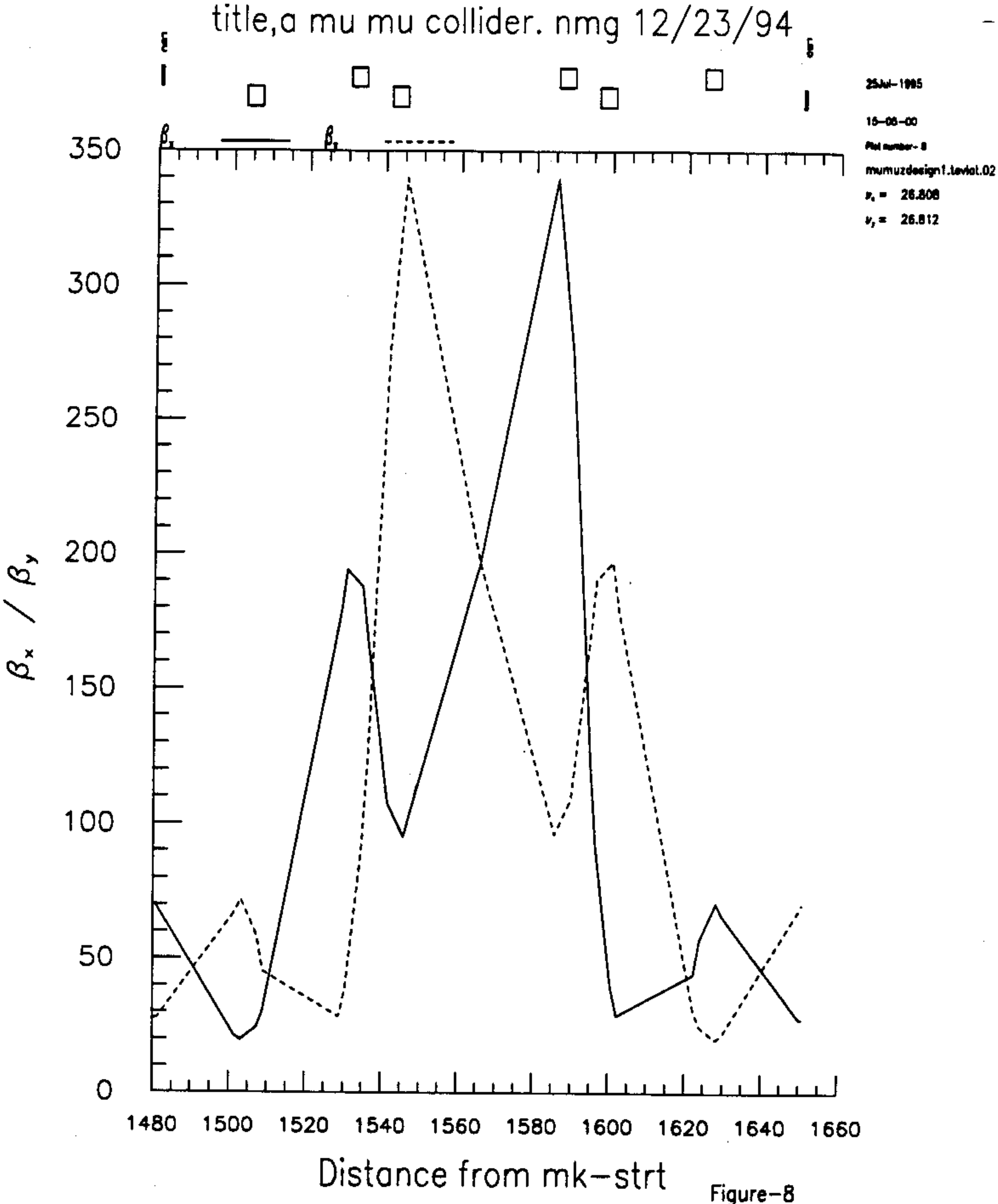
3250
3275
3300
3325
3350
3375
Distance from mk-strt

Figure-7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β_y
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250
200
150
100
50
0
β_x / β_y
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1500
1520
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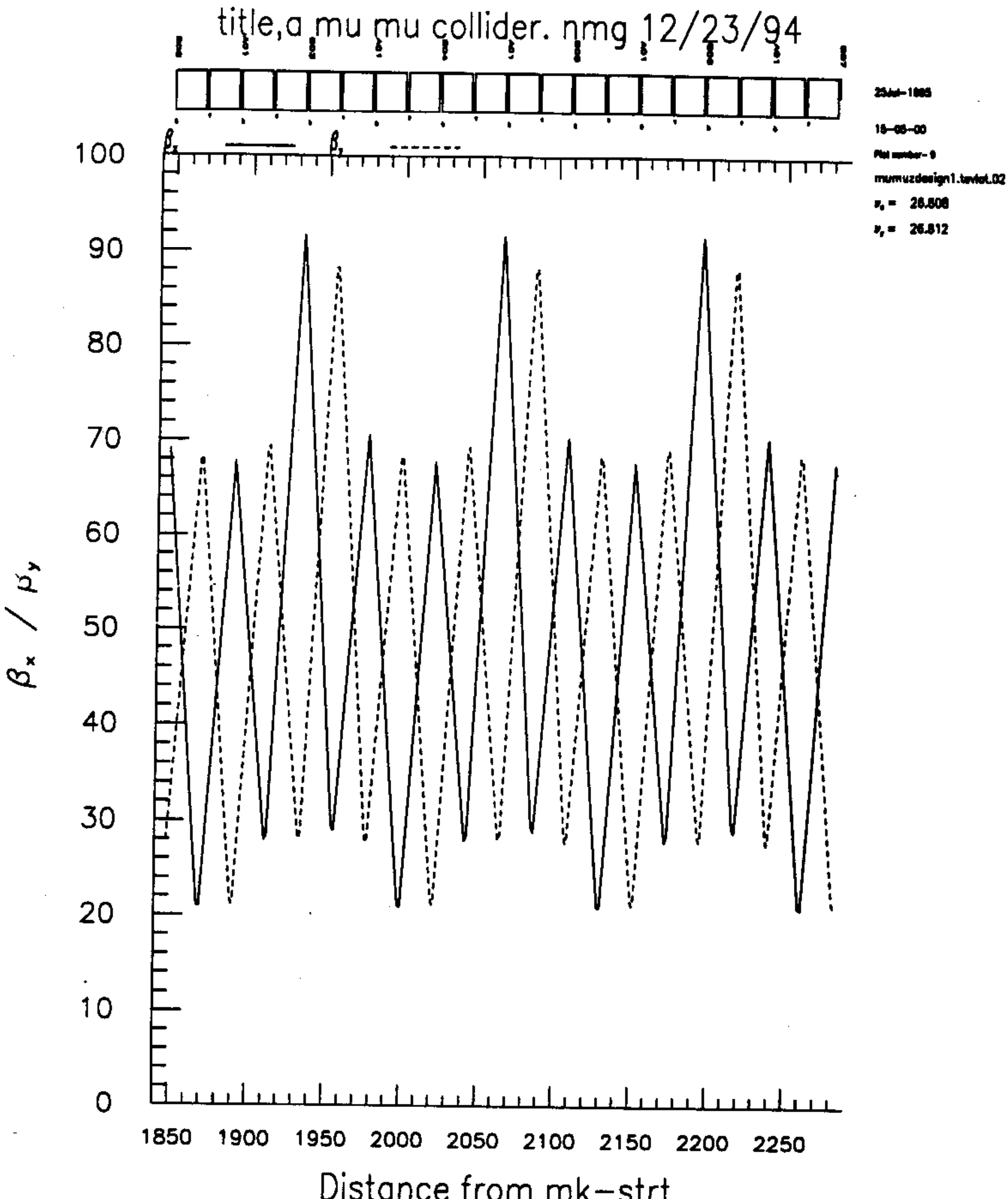
1560
1580
1600
1620
1640
1660
Distance from mk-strt

Figure-8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β_y
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β_x / β_y
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20
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40
50
60
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90
100
1850
1900
1950
2000
2050
2100
2150
2200
2250
Distance from mk−strt

Figure−9